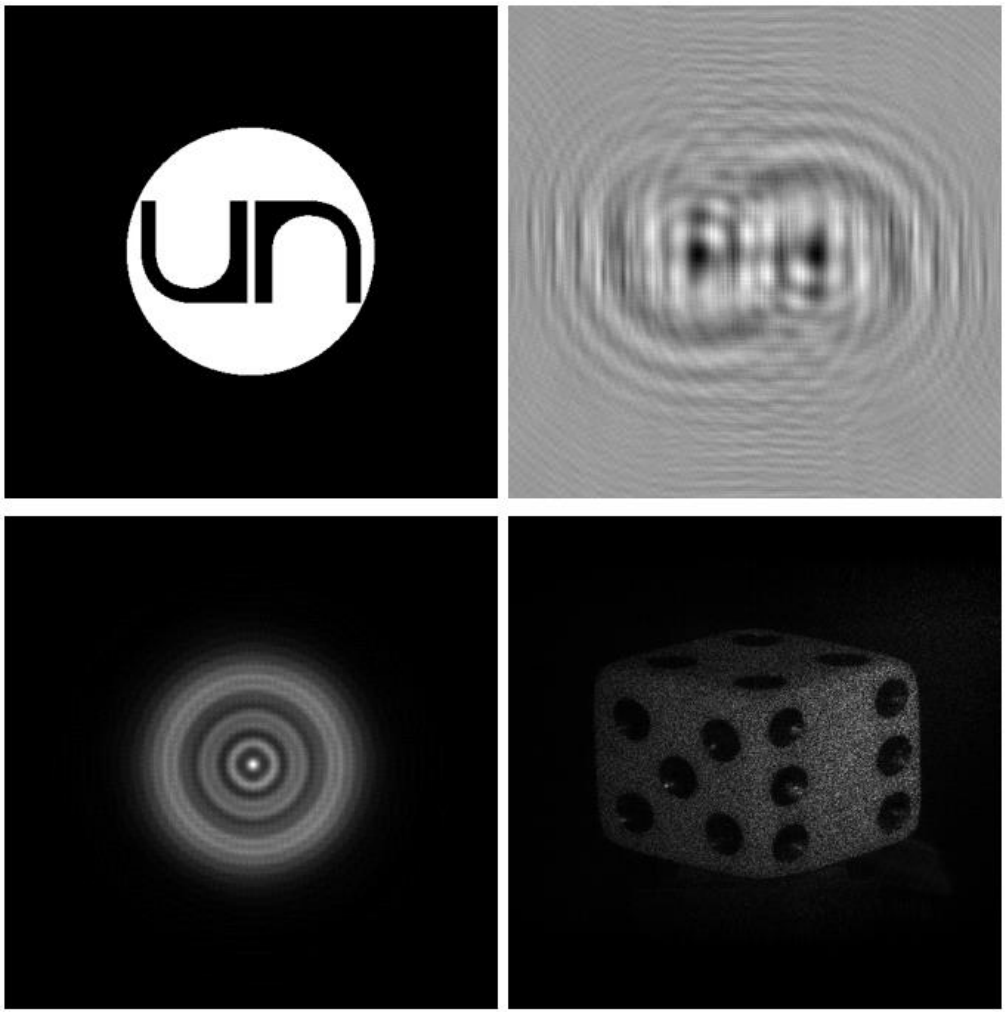
Numerical Propagation Manual



Numerical Propagation Manual – Version 1.2

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1.1 Introduction

Numerical Propagation is a plugin developed to work on the well-known software for image processing ImageJ. This plugin enables the computing of numerical wave propagation through the use of angular spectrum, Fresnel and Fresnel-Bluestein algorithms. The plugin can be used for teaching and research purposes.

1.2 Installation

The installation of the plugin can be done, following this steps:

1. Make sure you are running ImageJ 1.48s or superior. The installation procedure for ImageJ can be found at http://imagej.nih.gov/ij/docs/install/.
2. Download the *.zip* file, which includes all the needed libraries, available on the *Downloads* section at http://unal-optodigital.github.io/NumericalPropagation.
   Extract the contents of the *zip* file in the *imagej/plugins* folder.

1.3 Changes

Version 1.2

- Removed 8-bit scaling for real and imaginary parts.
- The plugin now works properly with images having an odd number of pixels.
- Added wavefront generation utilities.

Version 1.1

- Complex inputs (real and imaginary parts) are now supported.
- Added the possibility of displaying real and imaginary parts of the output field.
- Added the relation lock function for Fresnel-Bluestein output sizes.
- Settings window reorganized.
- Added the option to enable or disable spatial filtering before propagation.
- Some visual changes.

Version 1.0

- Initial release.

1.4 Support

If you are using Numerical Propagation and find a bug, please contact us at jppiedrahitaq@unal.edu.co or racastanedaq@unal.edu.co.

1.5 Credits

Numerical Propagation uses the diffraction algorithms available in JDiffraction (http://unal-optodigital.github.io/JDiffraction/), the fast and reliable FFT routines of JTransforms (https://sites.google.com/site/piotrwendykier/software/jtransforms/) and the icons provided in the Silk icon set (http://www.famfamfam.com/lab/icons/silk/).

1.6 Citation

You can reference Numerical Propagation using the following papers:

1. P. Piedrahita-Quintero, R. Castañeda, and J. Garcia-Sucerquia, "Numerical wave propagation in ImageJ," Appl. Opt. 54, 6410 (2015). doi: 10.1364/AO.54.006410
2. R. Castañeda, P. Piedrahita-Quintero, and J. Garcia-Sucerquia, "Image processing and computing for digital holography with ImageJ," Opt. Pura y Apl. 48, 77–84 (2015). doi: 10.7149/OPA.48.2.77

2 Running the plugin

To start working with the Numerical Propagation plugin, ImageJ must be running, if you don't know how to do so, please refer to the ImageJ user guide. If the Numerical Propagation plugin is installed, a new menu entry (*OD*) should appear on the ImageJ's main window, as shown in Figure 1. To run the plugin, just click the *Numerical Diffraction* entry and the main window should be displayed.

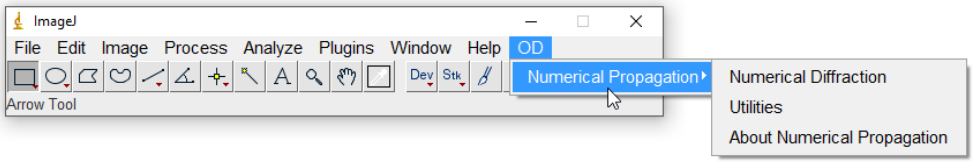

Figure 1. ImageJ main menu. See the OD label under which is the Numerical Propagation plugin.

2.1 Main window

The main window of the plugin is show in Figure 2. The graphical interface can be divided in four panels, parameters (1), log (2), outputs (3) and buttons (4).

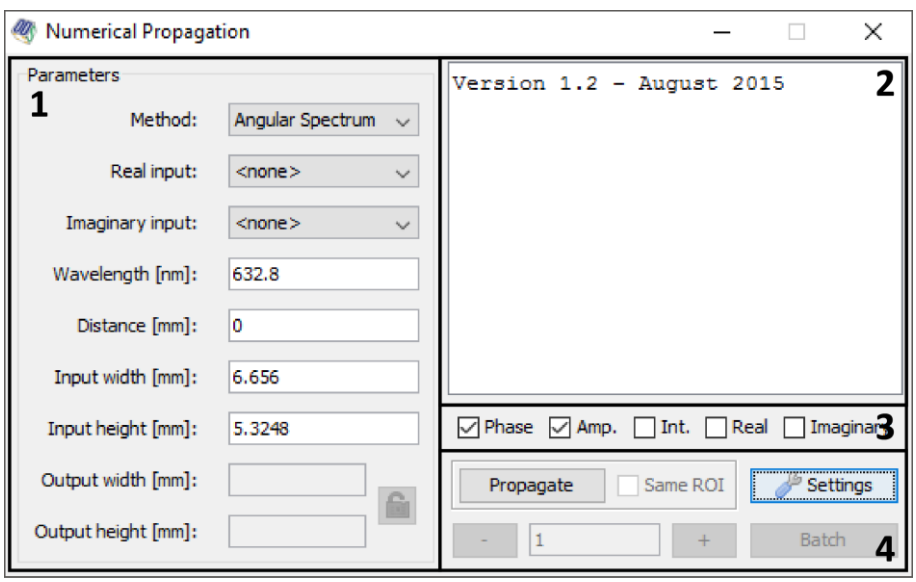

Figure 2. Main window.

2.1.1 Parameters

Method: Method to perform the numerical propagation. Can be picked between angular spectrum, Fresnel-Fourier, Fresnel-Bluestein and Automatic; the latter chooses automatically between angular spectrum and Fresnel-Fourier according with the limits stablished on the literature.

Real/Imaginary Input: Image representation of the input field's real/imaginary part.

Wavelength: The illumination wavelength.

Distance: Distance between the input and output fields.

Input width/height: Width/height of the input plane.

Output width/height: Width/height of the output plane (Fresnel-Bluestein only).

2.1.2 Log

Recent history of the plugin usage. When a propagation operation is performed, the parameters (method, input images, dimensions, wavelength, etc.) are printed in this text area. The log can be cleared right-clicking the area and selecting the *Clear* option.

2.1.3 Outputs

Possible representations of the output field. When the input field is propagated each one of the selected representations are shown in separate windows.

2.1.4 Buttons

Propagate: Performs the diffraction operation according with the selected parameters and settings.

Same ROI: Only available when the spatial filtering option is enabled. If this option is checked, the propagation is done using the lastly selected set of spatial frequencies.

Settings: Shows the settings window.

- / +: Performs the diffraction operation using the lastly selected set of spatial frequencies to a distance equals to the last used plus or minus the step given in the text field between the buttons.

Batch: Shows the batch propagation window.

3 Propagation (Working with the plugin)

The plugin allows the calculation of the numerical diffraction to be done in different ways, but in all cases, it can be resumed in three simple steps, i) *Input:* reading of the input parameters, ii) *Diffraction:* propagation of the input field, iii) *Output:* display of the selected representations.

3.1 Methods

The plugin has three algorithms to perform the numerical propagation of the input field, angular spectrum, Fresnel-Fourier and Fresnel-Bluestein. In the later subsections a brief introduction to the methods is given. Without losing generality, the expressions are presented in its one-dimensional form.

On the expressions in the following subsections, $k = 2\pi / \lambda$ stands for the wave number, λ for the wavelength, Δx / $\Delta \xi$ for the input/output sampling pitches, M for the number of data points, z for the distance, i for the imaginary unit and $U_n(0)$ for the complex field at the input plane.

3.1.1 Angular spectrum

This method does not include any approximations and is suitable for calculation of diffraction at short distances (µm - cm). Its FFT implementation restricts the input and output planes dimensions to be equal sized. The FFT implementation of the angular spectrum method is given by,

$$U_p(z) = \mathrm{IFFT}\left\{\mathrm{FFT}\left\{U_n(0)\right\}\exp\left(iz\sqrt{k^2 - 4\pi^2\left(\Delta f_x\right)^2 n^2}\right)\right\}$$

where

$$\Delta f_x = \frac{1}{M\Delta x}$$

3.1.2 Fresnel - Fourier

Fresnel diffraction can be obtained from the diffraction integral using the paraxial approximation. Its FFT implementation is given by,

$$U_p(z) = \Delta x \frac{\exp(ikz)}{i\lambda z}\exp\left(\frac{ik}{2z}\left(p\Delta\xi\right)^2\right)\mathrm{FFT}\left\{U_n(0)\exp\left(\frac{ik}{2z}\left(n\Delta x\right)^2\right)\right\}$$

Fresnel diffraction in its Fourier form fixes the output pitches to

$$\Delta\xi = \frac{\lambda z}{M\Delta x}$$

3.1.3 Fresnel - Bluestein

As above-mentioned, Fresnel-Fourier method introduces a restriction on the output pitches size. This restriction can be removed rewriting the product $2np$, present in the kernel of the Fourier transform $\exp(-i2\pi np / M)$, as $2np = n^2 + p^2 - (p-n)^2$. Performing this substitution, introduced by Bluestein, and reorganizing the terms, the Fresnel-Fourier expression can be turned into a circular convolution, given by,

$$U_p(z) = \frac{\exp(ikz)}{i\lambda z}\exp\left(-\frac{ik}{2z}\Delta\xi(\Delta x - \Delta\xi)p^2\right)\mathrm{IFFT}\{F_{1n}F_{2n}\}$$

with F_{1n} and F_{2n} given by

$$F_{1n} = \mathrm{FFT}\left\{U_n(0)\exp\left(\frac{ik}{2z}\Delta x(\Delta x - \Delta\xi)n^2\right)\right\}$$

$$F_{2n} = \mathrm{FFT}\left\{\exp\left(\frac{ik}{2z}\Delta x\Delta\xi n^2\right)\right\}$$

3.1.4 Automatic

The *Automatic* option, chooses between angular spectrum and Fresnel – Fourier according to the limits of use described in the literature. There is a critical distance (Z_c) given by the input field dimension and the wavelength.

$$Z_c = \frac{M\Delta x^2}{\lambda}$$

If the propagation distance is greater than or equal to Z_c , Fresnel-Fourier method is used, otherwise angular spectrum is used.

3.2 Single propagation

Single propagation is the only form of propagation that allows the spatial filtering, making it the basis for all the other options, this means, the other possibilities are only available after a single propagation is performed.

A single propagation is performed when the *Propagate* button is clicked, after this, the plugin reads the input parameters from the text fields on the main window and performs the calculations over the input field. If the spatial filtering option is enabled, the calculations are done over the filtered field.

When the *Same ROI* checkbox is selected, the input field is filtered using the parameters of the lastly selected set of spatial frequencies.

3.2.1 Spatial filtering

When a single propagation is going to be done and the spatial filtering option is enabled, two new windows are shown, the first showing the Fourier spectrum of the input field, and the second, a filtering window, see Figure 3, where you can choose between manual and coordinate filtering.

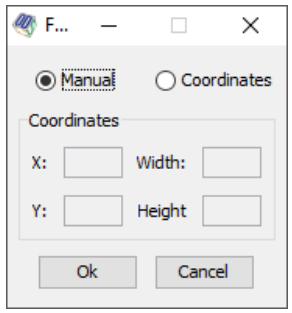

Figure 3. Filtering window.

Manual filtering

One interesting feature of the plugin, is that it allows the free selection of the set of spatial frequencies through the ImageJ selection tools (*Rectangular*, *Oval*, *Polygon* and *Freehand*). To perform *Manual* filtering, click the Manual radio button, select the region of interest (ROI) on the Fourier spectrum and click the *Ok* button.

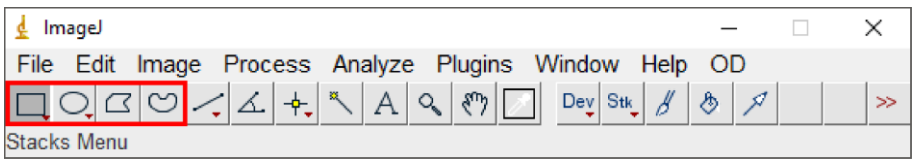

Figure 4. ImageJ selection tools.

Coordinates filtering

Coordinates filtering selects the ROI on the spatial frequencies spectrum using a Rectangular ROI, given by the parameters introduced on the text fields in the Coordinates panel. To perform the *Coordinates* filtering, click the Coordinates radio button, introduce the parameters and click the *Ok* button.

3.3 Step propagation

Step propagation can be done through the + and - buttons present on the main window. This type of propagation is only available after a single propagation is performed, because it diffracts the last used complex field (filtered or not, according with the selected settings). The field is propagated to a distance equals to the last used distance plus or minus the step given in the text field between the buttons.

To perform a step propagation, introduce the step value in the text field using the same units as the distance parameter, then click the + or - button.

3.4 Batch propagation

Batch propagation is available through the *Batch* button present on the main window. Batch propagation consist of a distance sweep, given by starting and ending distances, and the number of planes or the step size. As in step propagation, the complex field to be propagated, is the lastly used input field (filtered or not).

4 Settings

The plugin offers a *Settings* window to customize the operation of the plugin, this window is accessible through the Settings button on the main window. As can be seen in Figure 5, this window has three tabs (Units, Propagation and Scaling).

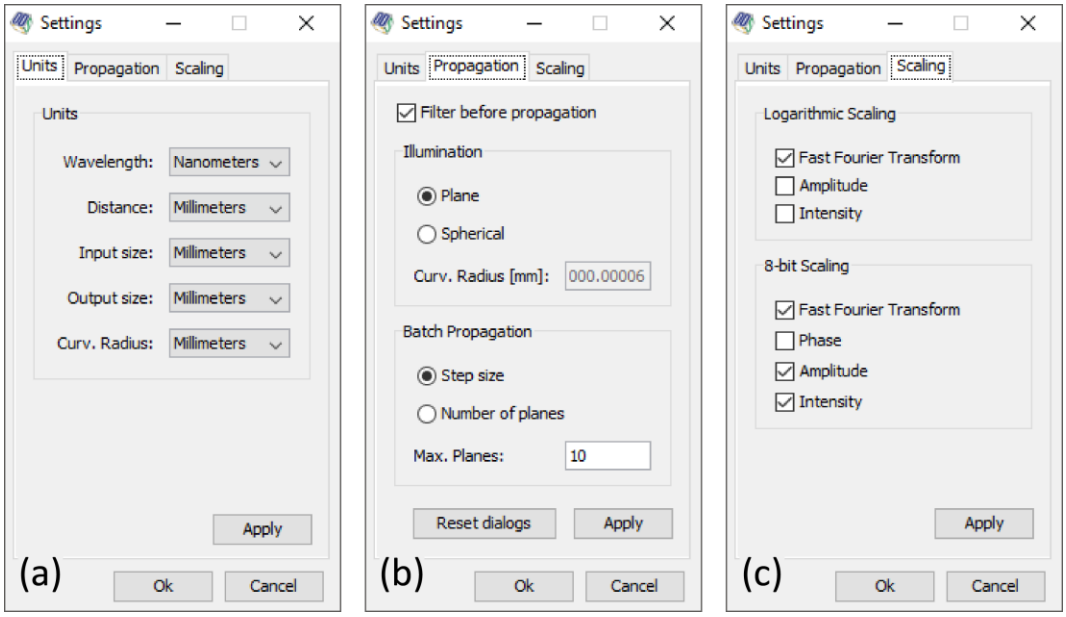

Figure 5. Settings window. (a) ***Units*** *tab. (b)* ***Propagation*** *tab. (c)* ***Scaling*** *tab.*

4.1.1 Units

On the units tab, the units of the parameters can be changed. The options offered are nanometers (nm), micrometers (µm), millimeters (mm), centimeters (cm) and meters (m).

4.1.2 Propagation

On the propagation tab, can be enabled or disabled the spatial filtering operation. The other options correspond to the illumination and the batch propagation.

Illumination

Illumination can be chosen between spherical and plane waves. If the spherical wave is selected as the illumination, the curvature radius can be set using the text field.

Batch Propagation

In this panel, can be selected the parameters for batch propagation. If you want to use the batch propagation tool by means of the step size or the number of planes, select the corresponding radio button.

The *Max. Planes* option configures the number of planes from which the plugin shows a warning message. This option is useful to avoid the propagation of a lot of planes by mistake, which can cause the freezing of ImageJ.

4.1.3 Scaling

In this tab, the options related to the scaling of the image representations can be found. It offers the option to display the images in logarithmic and 8-bit scaling (0 - 255).

5 Utilities

The utilities tool bundled with the plugin is available through the *Utilities* label, under the *OD>Numerical* Propagation menu, see Figure 1. The main purpose of this tool is to offer the possibility to do complex math with operands in the form of images. In the new version this tool has been extended and now includes wavefront generation functions. The utilities window is shown in Figure 6

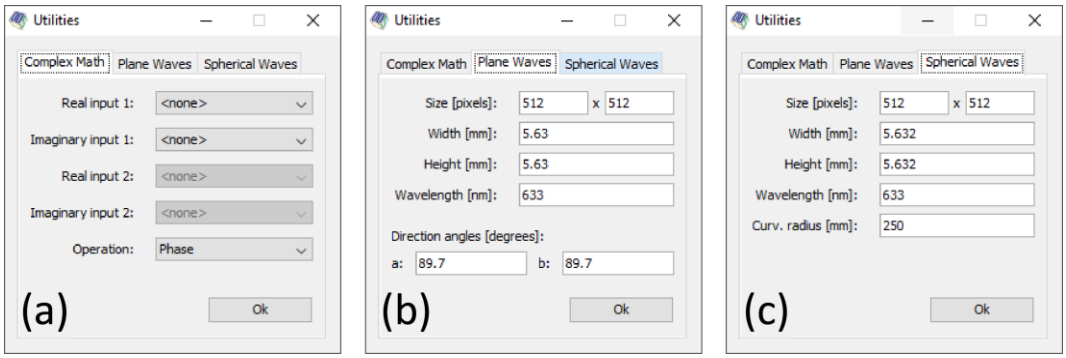

Figure 6. Utilities window.

5.1 Complex math

The possible operations are *Phase*, *Amplitude*, *Intensity*, *Addition*, Subtraction and *Multiplication*. In the case of *Phase*/*Amplitude*/*Intensity* the result of the operation is the *Phase*/*Amplitude*/*Intensity* of the complex field given by the inputs on the *Real/Imaginary input 1* combo boxes, see Figure 6 (a)**¡Error! No se encuentra el origen de la referencia.**. On the remaining, the result is given by the pointwise operation between the two complex input fields selected on the combo boxes.

5.2 Plane waves

Figure 6 (b) shows the Plane wavefronts generation tab. The complex field is calculated using a general expression for a plane wave

$$R(x,y) = A(x,y)\exp\left[ik\left(x\cos a + y\cos b + z\cos c\right)\right]$$

where i is the imaginary unit, k is the wave number and a, b and c are the director angles measured as shown in Figure 7.

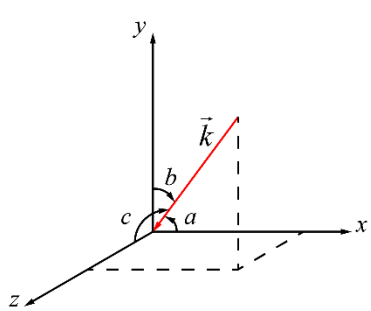

Figure 7. Direction angles for plane waves.

The parameters required for the calculation are explained below.

Size: Dimension in pixels of the images that are going to be generated.

Width/height: Physical dimensions of the field.

Wavelength: Wavelength of the complex wavefront.

Direction angles: Direction angles measured as shown in Figure 7. Only a and b are required, c is calculated using $\cos^2 a + \cos^2 b + \cos^2 c = 1$.

5.3 Spherical waves

For divergent spherical wavefronts generation the required parameters are identical to the ones in the plane wavefields, except for the *Curvature radius*, this refers to the distance between the point source and the plane where the complex field is going to be generated.

6 Usage examples

6.1 Reconstruction of digital holograms

The reconstruction of digital holograms can be understood as the diffraction process that a wave experiments when it illuminates the digitally recorded hologram, therefore the Numerical Propagation plugin can be used for this purpose. A step-by-step guide to reconstruct a digital hologram is described below.

1. Open the digital hologram image file.
2. Select the opened image in the *Real input* combo box.
3. Select the method of propagation in the *Method* combo box. The method should be selected taking into account the propagation distance.
4. Introduce the parameters, *Wavelength*, *Distance*, *Input width/height*, in the corresponding text fields. In case that the Fresnel - Bluestein method is selected, also introduce the *Output width/height*.
5. Select the representations of the output field.
6. Click on the *Propagation* button. It is recommended to perform a spatial filtering operation on the input hologram to remove undesired information.
7. If the spatial filtering option is enabled, select the ROI on the Fourier spectrum and click on the *Ok* button of the filtering window. See spatial filtering for further information.

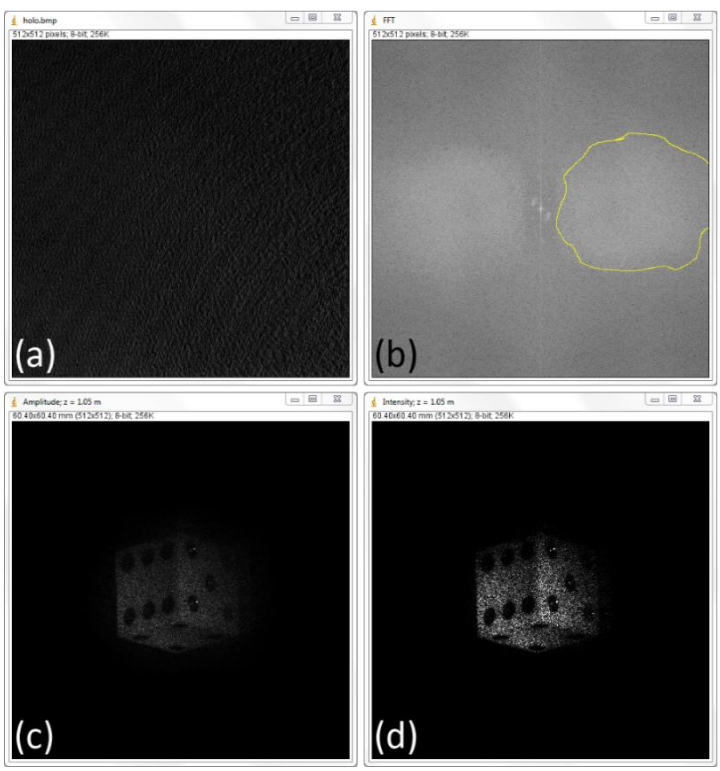

Figure 8. Reconstruction of a digital hologram. (a) Digital hologram. (b) Selected ROI on the Fourier spectrum. (c) Amplitude representation of the reconstructed hologram. (d) Intensity representation of the reconstructed hologram, with enhanced brightness and contrast.