Inspira Crea Transforma

UNIVERSIDAD EAFIT®

Welfare gains of the poor: An endogenous Bayesian approach with spatial random effects

Andrés Ramírez Hassan

Universidad EAFIT

Santiago Montoya Blandón

Universidad EAFIT

Seminario Institucional: Escuela de Estadística
Universidad Nacional sede Medellín

February 16, 2015

Outline

Outline

Novelty

- Bayesian simultaneous equations system with spatial random effects suited to handle spatial dependence, heterogeneity, endogeneity and statistical inference associated with complicated non-linear functions of parameter estimates.
- Full conditional posterior distributions available along with Gibbs sampler implementation.

Application

Evaluate welfare implications on poor households, measured through the Equivalent Variation, caused by the electricity price changes in the province of Antioquia (Colombia), after EPM acquired EADE in 2006. Spatial effects, endogeneity between price and electricity demand, unobserved heterogeneity and statistical inference on the Equivalent Variation are all considered. Important motivation for the application: electricity services represent a significant share of households' budget, and this fact is prominent on the poor population (Gomez-Lobo, 1996, Ruijs, 2009, You and Lim, 2013).

Methodology

- Spatial effects to perform statistical inference based on cross-sectional areal data have a long tradition in spatial statistics (Cressie, 1993, Ripley, 2004, Anselin, 1988).
- Mostly Frequentist with some noteworthy exceptions founded on Bayesian methods (LeSage, 1997, 2000, Parent and LeSage, 2008, LeSage and Pace, 2009, LeSage and Llano, 2013).
- Endogeneity has been treated from the spatial perspective (SAR), and recently from the regressors (Rey and Boarnet, 2004, Kelejian and Prucha, 2004, Fingleton and Gallo, 2008, Crukker et al., 2013, Liu and Lee, 2013).

Methodology

- Unobserved heterogeneity has been tackled using a Bayesian approach. References involve simultaneously spatial effects and unobserved heterogeneity, but they do not take into consideration recursive endogeneity (LeSage, 2000, Smith and LeSage, 2004, LeSage et al., 2007, Parent and LeSage, 2008, Seya et al., 2012, LeSage and Llano, 2013).

Applications

- Microeconomic foundation has been developed to great extent but not completely using a logarithmic demand function in an increasing-block pricing scheme.
- Welfare implications based on estimates that account for spatial characteristics, endogeneity and unobserved heterogeneity are scarce in literature.
- Acton and Mitchell (1983), Gomez-Lobo (1996), Dodonov et al. (2004), Lundgren (2009), Ruijs (2009), You and Lim (2013) examine welfare implications from price changes in utilities. All impacts on lowest income groups are found to be considerable, even those associated with small changes.

Outline

Characteristics

General characteristics of the Colombian energy market that are key to understanding the price variations we seek to analyze:

- Division by socioeconomic strata.
- Regulator establishes subsidized subsistence electricity consumption for strata one, two and three. Subsistence consumption differs depending on elevation.
- Each company has a reference tariff over its market.
- Generation, transport at country level, distribution at market level and commercialization define tariffs. Same reference tariff is not equal to same average prices for strata and municipalities.

Tariff unification process

- EPM acquires EADE in 2006.
- EPM serves an urban market with large population density, EADE attended rural areas with low population density.
- First: low tariffs for EPM's market, high tariffs for EADE's market. Now: huge decreases in tariffs for rural areas (up to 17.53%), slight increase on urban areas.
- Welfare implications are considerable for rural municipalities and poorest households.

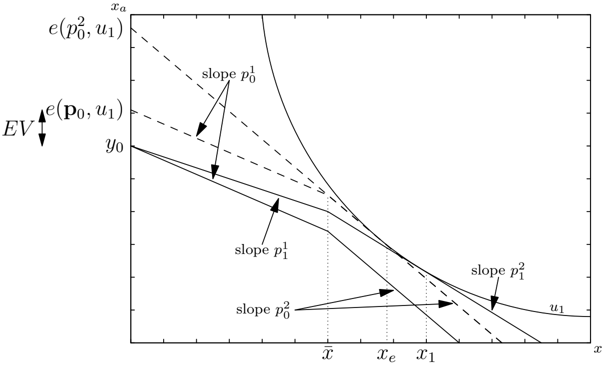
Outline

Equivalent Variation

- There is Consumer Surplus (CS), Compensating Variation (CV) and Equivalent Variation (EV). Which one to pick?
- CS leaves income effects out but they are implied in the problem. CV does not rank alternatives well if preferences are not homothetic and income changes. EV encompasses both.
- Increasing-block prices create non-linear budget constraints (see Figure 1)

Equivalent Variation

Figure 1: Example of Equivalent Variation with price changes on both tiers, and new and virtual consumption on the second tier



Equivalent Variation

From Varian (1992), EV can be expressed in terms of expenditure functions as

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = e(\mathbf{p}_0, u_1) - e(\mathbf{p}_1, u_1) = e(\mathbf{p}_0, u_1) - y_0 \quad (1)$$

Subscripts represent time, superscripts the block, $\bar{x}$ subsistence consumption. We consider two cases:

(i) $u_1 = V((p_1^1, 1), y_0)$

(ii) $u_1 = V((p_1^2, 1), y_0 + (p_1^2 - p_1^1)\bar{x})$

Equivalent Variation

For (i):

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = e(p_0^1, u_1) - y_0 \tag{2}$$

For (ii):

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = e(p_0^2, u_1) - (p_0^2 - p_0^1)\bar{x} - y_0 \tag{3}$$

Hausman (1981) developed a method that relates the econometric specification of a Marshallian demand function to the definitions stated above. We model the demand as:

$$x(p, y) = p^{\alpha} y^{\delta_1} e^{z'\delta} \tag{4}$$

Equivalent Variation

The method shown by Hausman (1981) leads to the following expressions for the EV. For (i):

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = \left[\frac{1-\delta_1}{1+\alpha}\left(p_0^{1(1+\alpha)} - p_1^{1(1+\alpha)}\right)e^{z'\boldsymbol{\delta}} + y_0^{1-\delta_1}\right]^{\frac{1}{1-\delta_1}} - y_0 \tag{5}$$

For (ii):

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = \Bigg[\frac{1-\delta_1}{1+\alpha}\left(p_0^{2(1+\alpha)} - p_1^{2(1+\alpha)}\right)e^{z'\boldsymbol{\delta}} + \left(y_0 + (p_1^2 - p_1^1)\bar{x}\right)^{1-\delta_1}\Bigg]^{\frac{1}{1-\delta_1}} - (p_0^2 - p_0^1)\bar{x} - y_0 \tag{6}$$

Outline

Model

We propose an endogenous Bayesian approach using simultaneous equations with spatial random effects to estimate a model facing recursive endogeneity (as an instrumental variable problem). The specification of our model is:

$$\ln x_i = \mathbf{z}'_{1i}\boldsymbol{\delta} + \alpha \ln p_i + u_{1i} \quad (7)$$

$$\ln p_i = \mathbf{z}'_{1i}\boldsymbol{\phi} + \alpha_s z_{2i} + u_{2i} \quad (8)$$

where u_{1i} and u_{2i} are the idiosyncratic error terms associated with the demand and price of each municipality, respectively. Following Greenberg (2008), we assume that $(u_{1i}, u_{2i})' \sim \mathcal{N}(\mathbf{0}, \Sigma)$, $\Sigma = \{\sigma_{ij}\}$, such that σ_{12} captures the endogeneity of the system.

Model

The previous model can be estimated from the reduced form equations:

$$\begin{aligned}\ln x_i &= \pi_0 + \pi_1 \ln y_i + \pi_2 \ln p_i^s + \pi_3 alt_i + \pi_4 \ln urb_i + \pi_5 EADE_i + \\ &\quad \mu_{1i} + v_i \\ \ln p_i &= \phi_0 + \phi_1 \ln y_i + \phi_2 \ln p_i^s + \phi_3 alt_i + \phi_4 \ln urb_i + \alpha_s EADE_i + \\ &\quad \mu_{2i} + v_i\end{aligned} \tag{9}$$

Model

Where $\mu_{1i} = u_{1i} + \alpha u_{2i}$ and $\mu_{2i} = u_{2i}$, such that $(\mu_{1i}, \mu_{2i})' \sim \mathcal{N}(\mathbf{0}, \Omega)$,

$$\Omega = \begin{bmatrix} \sigma_1^2 + \alpha^2\sigma_2^2 + 2\alpha\sigma_{12} & \sigma_{12} + \alpha\sigma_2^2 \\ \sigma_{12} + \alpha\sigma_2^2 & \sigma_2^2 \end{bmatrix} = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{bmatrix}$$

and v_i captures the spatial interactions between neighboring municipalities.

Likelihood

Setting $\mathbf{z}_i' = (\ln y_i, \ln p_i^s, alt_i, \ln urb_i, EADE_i)$,
$\boldsymbol{\pi}' = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$, $\boldsymbol{\phi}' = (\phi_1, \phi_2, \phi_3, \phi_4, \alpha_s)$ and
$\mathbf{v}' = (v_1, v_2, \ldots, v_n)$.
The likelihood function, $f(\ln x, \ln \mathbf{p} | \Omega, \boldsymbol{\pi}, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v})$, *is* :

$$\frac{|\Omega|^{-N/2}}{(2\pi)^{N/2}} \exp\left\{ -\frac{1}{2} \sum_{i=1}^{N} (\ln x_i - \mathbf{z}_i'\boldsymbol{\pi} - \pi_0 - v_i, \ln p_i - \mathbf{z}_i'\boldsymbol{\phi} - \phi_0 - v_i)\Omega^{-1} \begin{pmatrix} \ln x_i - \mathbf{z}_i'\boldsymbol{\pi} - \pi_0 - v_i \\ \ln p_i - \mathbf{z}_i'\boldsymbol{\phi} - \phi_0 - v_i \end{pmatrix} \right\}$$

Prior: CAR Process

We assume the spatial random effect follows as prior distribution an improper (intrinsic) Conditionally Autoregressive (CAR) structure (Besag et al., 1991):

$$v_i | \mathbf{v}_{i\sim j} \sim \mathcal{N}\left(\sum_{i\sim j} \frac{w_{ij} v_j}{\sum_{i\sim j} w_{ij}}, \frac{\sigma_v^2}{\sum_{i\sim j} w_{ij}}\right)$$

The joint distribution of the improper CAR is $\mathbf{v} \sim \mathcal{N}(\bar{\mathbf{v}}, \sigma_v^2(\mathbf{I_N} - \mathbf{W_n})^{-1})$ where W_n is the contiguity matrix (Wall, 2004).

Prior: CAR Process

Why CAR over SAR? Literature can be found in favor of each one (Banerjee et al., 2004, Parent and LeSage, 2008, Darmofal, 2009, Chakraborty et al., 2013, for the CAR) and (Smith and LeSage, 2004, LeSage et al., 2007, LeSage and Llano, 2013, for the SAR). Our decision is based on:

- Inherent heteroscedasticity and therefore higher levels of heterogeneity (Cressie, 1993).
- Markovian characteristics in space. Spatial heterogeneity is due to local variation, rather than a global spatial pattern, as in the SAR (Anselin, 2003).
- Computational convenience and intuitiveness (Banerjee et al., 2004).

Prior: Location Parameters

To complete our Bayesian specification, we set the other priors as follow: the elasticities and semi-elasticities have a Multivariate Normal distributions with mean vector $\mathbf{0}$ and precision matrices $0.001 I_5$. This implies vague prior information where there is no effect of each control variable on dependent variables.

Prior: Scale Matrix

In addition, we assume a Wishart distribution for Ω^{-1} with 3 degrees of freedom and scale matrix I_2. Setting the degrees of freedom to $p+1$, where p is the dimension of the covariance matrix, the Wishart form reduces to $\pi(\Omega^{-1}) \propto |\Omega^{-1}|^{-(N+1)/2}$, which is a diffuse prior used by Savage that emerges using Jeffrey's invariance theory (Zellner, 1996). Thus, a priori there is no endogeneity, and the fat-tailed prior will guarantee robustness of outcomes regarding this prior distribution (Berger, 1985).

Prior: Scale parameter

Finally, $1/\sigma^2_{v_i}$, which is the precision parameter of the CAR component, has a prior Gamma distribution with shape and scale parameters equal to 0.5 and 0.0005, respectively. This parametrization is frequently found in literature (Kelsall and Wakefield, 1999, Thomas et al., 2007), and expresses the prior belief that the standard deviation for the spatial random effects is centered around 0.05 with a 1% prior probability of being smaller than 0.01 or larger than 2.5.

Prior: Scale parameter

Those specific values of the hyperparameters are inspired in the fact that we have two different sources of stochastic variability in our model, $\mu_{il}, \{l = 1, 2\}$ and (v_i). As a consequence, both sets of hyperparameters of the prior distributions of these random effects cannot imply arbitrarily large variability, since these effects would be unidentifiable; we try to identify two random effects using a single observation at each spatial unit. Thus, it is necessary to fix the hyperparameters of the Gamma distribution, given that we use a diffuse prior distribution for Ω^{-1} (Banerjee et al., 2004).

Posterior: Scale Matrix

$\pi(\Omega|\boldsymbol{\pi}, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v}, \sigma_v^2, Data) \sim \mathcal{W}_\kappa(\bar{\omega}, \bar{\Omega})$, where $\bar{\omega} = \underline{\omega} + N$ and

$$\bar{\Omega} = \left[\underline{\Omega}^{-1} + \sum_{i=1}^{N} \begin{pmatrix} \ln x_i - \mathbf{z}_i'\boldsymbol{\pi} - \pi_0 - v_i \\ \ln p_i - \mathbf{z}_i'\boldsymbol{\phi} - \phi_0 - v_i \end{pmatrix} (\ln x_i - \mathbf{z}_i'\boldsymbol{\pi} - \pi_0 - v_i, \ln p_i - \mathbf{z}_i'\boldsymbol{\phi} - \phi_0 - v_i)\right]^{-1}$$

Posterior: Location parameters demand equation

To sample $\boldsymbol{\pi}$, we use $f(\ln x_i, \ln p_i|\boldsymbol{\Theta}) = f(\ln x_i|\ln p_i, \boldsymbol{\Theta})f(\ln p_i|\boldsymbol{\Theta})$ where $\boldsymbol{\Theta} = (\Omega, \boldsymbol{\pi}, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v})$.

In particular,

$\ln x_i|\ln p_i, \boldsymbol{\Theta} \sim \mathcal{N}(\mathbf{z}_i'\boldsymbol{\pi} + \pi_0 + v_i + \frac{\omega_{12}}{\omega_{22}}(\ln p_i - \mathbf{z}_i'\boldsymbol{\phi} - \phi_0 - v_i), \psi_{11})$ where $\psi_{11} = \omega_{11} - \frac{\omega_{12}^2}{\omega_{22}}$. In addition, the prior is $\mathcal{N}_k(\underline{\pi}, \underline{\Pi})$.

Posterior: Location parameters demand equation

Then, $\pi(\boldsymbol{\pi}|\Omega, \phi, \pi_0, \phi_0, \mathbf{v}, \sigma_v^2, Data) \sim \mathcal{N}_k(\bar{\pi}, \bar{\Pi})$ where
$\bar{\Pi} = \left[\underline{\Pi}^{-1} + \psi_{11}^{-1}\sum_{i=1}^{N}\mathbf{z}_i\mathbf{z}_i'\right]^{-1}$ and
$\bar{\pi} = \bar{\Pi}\left(\underline{\Pi}^{-1}\underline{\pi} + \psi_{11}^{-1}\sum_{i=1}^{N}\mathbf{z}_i(\ln x_i - \frac{\omega_{12}}{\omega_{22}}(\ln p_i - \mathbf{z}_i'\phi - \phi_0 - v_i) - \pi_0 - v_i)\right)$.

Posterior: Location parameters price equation

We follow the same procedure to deduce the conditional posterior distribution of ϕ, that is, we use $f(\ln x_i, \ln p_i|\mathbf{\Theta}) = f(\ln p_i|\ln x_i, \mathbf{\Theta})f(\ln x_i|\mathbf{\Theta})$.
In particular,
$\ln p_i|\ln x_i, \mathbf{\Theta} \sim \mathcal{N}(\mathbf{z}_i'\phi + \phi_0 + v_i + \frac{\omega_{12}}{\omega_{11}}(\ln x_i - \mathbf{z}_i'\boldsymbol{\pi} - \pi_0 - v_i), \psi_{22})$
where $\psi_{22} = \omega_{22} - \frac{\omega_{12}^2}{\omega_{11}}$. Additionally, the prior is $\mathcal{N}_k(\underline{\phi}, \underline{\Phi})$.

Posterior: Location parameters price equation

Thus, $\pi(\phi|\boldsymbol{\pi}, \Omega, \pi_0, \phi_0, \mathbf{v}, \sigma_v^2, Data) \sim \mathcal{N}_k(\bar{\phi}, \bar{\Phi})$ where

$\bar{\Phi} = \left[\underline{\Phi}^{-1} + \psi_{22}^{-1} \sum_{i=1}^{N} \mathbf{z}_i \mathbf{z}_i'\right]^{-1}$ and

$\bar{\phi} = \bar{\Phi}\left(\underline{\Phi}^{-1}\underline{\phi} + \psi_{22}^{-1} \sum_{i=1}^{N} \mathbf{z}_i (\ln p_i - \frac{\omega_{12}}{\omega_{11}}(\ln x_i - \mathbf{z}_i'\boldsymbol{\pi} - \pi_0 - v_i) - \phi_0 - v_i)\right).$

Posterior: constant demand equation

Regarding the posterior distribution of the constant term π_0, using as prior an improper uniform distribution and given $f(\ln x_i, \ln p_i|\boldsymbol{\Theta}) = f(\ln x_i|\ln p_i, \boldsymbol{\Theta})f(\ln p_i|\boldsymbol{\Theta})$, we obtain that $\pi_0 \sim \mathcal{N}(\bar{\pi}_0, \psi_{11})$ where $\bar{\pi}_0 = \ln x_i - \frac{\omega_{12}}{\omega_{22}}(\ln p_i - \mathbf{z}_i'\boldsymbol{\phi} - \phi_0 - v_i) - v_i - \mathbf{z}_i'\boldsymbol{\pi}$.

Posterior: constant price equation

In a similar way, using as prior an improper uniform distribution for ϕ_0, and the fact that $f(\ln x_i, \ln p_i|\mathbf{\Theta}) = f(\ln p_i| \ln x_i, \mathbf{\Theta})f(\ln x_i|\mathbf{\Theta})$, we obtain that
$\phi_0 \sim \mathcal{N}(\bar{\phi}_0, \psi_{22})$ where
$\bar{\phi}_0 = \ln p_i - \frac{\omega_{12}}{\omega_{11}}(\ln x_i - \mathbf{z}_i'\boldsymbol{\pi} - \pi_0 - v_i) - v_i - \mathbf{z}_i'\phi$.

Posterior: spatial effects

Using the fact that $f(\ln x_i, \ln p_i|\mathbf{\Theta}) = f(\ln p_i| \ln x_i, \mathbf{\Theta}) f(\ln x_i|\mathbf{\Theta})$ to write the likelihood function and the improper CAR formulation, $\pi(v_i|\mathbf{v}_{-i}, \boldsymbol{\pi}, \Omega, \pi_0, \phi_0, \sigma_v^2, Data)$ is $\mathcal{N}(\bar{\xi}, \bar{\eta})$ where

$$\bar{\eta} = \left[\left(\frac{\sigma_v^2}{w_{i+}}\right)^{-1} + N\left(\frac{\psi_{22}}{\left(\frac{\omega_{12}}{\omega_{11}}+1\right)}\right)^{-1}\right]^{-1}$$ and $\bar{\xi}$ equal to

$$\bar{\eta}\left[\left(\frac{\sigma_v^2}{w_{i+}}\right)^{-1}\sum_{i\sim j}\left(\frac{w_{ij}}{w_{i+}}\right)v_j + \left(\frac{\psi_{22}}{\left(\frac{\omega_{12}}{\omega_{11}}+1\right)^2}\right)^{-1}\sum_{i=1}^{N}\left(\left(\frac{1}{\frac{\omega_{12}}{\omega_{11}}+1}\right)(\ln p_i - \frac{\omega_{12}}{\omega_{11}}(\ln x_i - \mathbf{z}_i'\boldsymbol{\pi} - \pi_0) - \phi_0 - \mathbf{z}_i'\boldsymbol{\phi})\right)\right]$$

Posterior: scale parameter

Given that $1/\sigma_v^2$ has as a prior $\mathcal{G}(\underline{\alpha}, \underline{\beta})$, its conditional posterior is $\mathcal{G}(\bar{\alpha}, \bar{\beta})$ where $\bar{\alpha} = \underline{\alpha} + 1/2$ and
$\bar{\beta} = \underline{\beta} + \left(\frac{w_{i+}}{2}\right)\left(v_i - \sum_{i\sim j}\left(\frac{w_{ij}}{w_{i+}}\right)v_j\right)^2$.

Outline

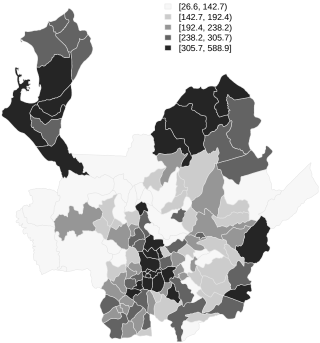
Descriptive Statistics

Table 1: Descriptive Statistics

Variable	Mean	Std. Dev.	Min	Max
Cons. (kWh)	234.874	117.811	26.595	588.937
Price (US$)	0.061	0.024	0.027	0.240
Income (US$)	397.085	95.242	230.514	619.227
Subs. Price (US$)	0.030	0.006	0.016	0.056
Altitude	29.032%	45.575%	0.000	1.000
Urbanization	45.876%	19.917%	10.700%	98.247%
Coverage	77.419%	41.981%	0.000	1.000

Source: Author's calculations

Figure 2: Average Annual Electricity Consumption per Household (kWh): Province of Antioquia (Colombia) in 2005, Stratum One



Model Estimation

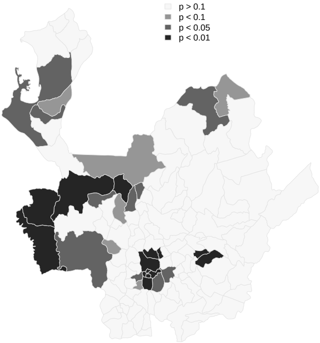
- Use of MCMC techniques, specifically Gibbs sampler algorithms. 10 million updates, 5 million discarded and draws ever 500.
- Robustness checks with different contiguity criteria: queen, rook and road length. Base model is road length, due to its economic sensibleness.
- Statistical inference can be performed and hypotheses can be tested thanks to the Bayesian approach.
- Convergence diagnostics show convergence of the chains and therefore validate the results.
- Global and local spatial dependence can be observed and is significant.

Table 2: Global spatial correlation tests

Road Length Contiguity		
Value	Moran I	Geary C
Statistic	-0.00032	0.82708
P-Value	0.02825	0.00098
Queen Contiguity		
Value	Moran I	Geary C
Statistic	0.37807	0.50262
P-Value	1.50e-13	2.07e-09
Rook Contiguity		
Value	Moran I	Geary C
Statistic	0.37717	0.50747
P-Value	3.15e-13	2.43e-09

Source: Author's calculations

Figure 3: Local Moran's I test p-values



Estimation Results

Table 3: Summary of structural parameter posterior estimates

Road Length Contiguity					
Parameter	Mean	Median	90% HPD Interval		$R_{01} = \frac{P(\theta \in (0,\infty))}{P(\theta \in (-\infty,0]))}$
			Lower	Upper	
Constant	1.913	1.940	-1.539	5.393	4.663
Price	-0.886	-0.882	-1.449	-0.278	0.014
Income	0.301	0.297	-0.054	0.636	11.920
Subs. Price	0.123	0.120	-0.215	0.449	2.560
Altitude	0.139	0.137	-0.041	0.304	9.235
Urbanization	0.571	0.566	0.410	0.724	908.090

Source: Author's calculations

Convergence Diagnostics

Table 4: Stationarity and Convergence diagnostics

Road Length Contiguity				
Parameter	Heidelberger (1st Part/p-value)[a]	Heidelberger (2nd Part)[b]	Geweke[c]	Raftery[d]
Constant	0.887	0.064	0.758	1.46
Price	0.923	-0.047	0.820	1.10
Income	0.414	0.035	-0.740	2.74
Subs. Price	0.909	0.067	-0.311	1.11
Altitude	0.581	0.052	-0.515	1.08
Urbanization	0.871	0.024	-0.407	1.05

Notes: [a] Null hypothesis is stationarity of the chain, [b] Half-width to mean ratio, [c] Mean difference test z-score, [d] Dependence factor

Main Results

Table 5: Equivalent Variation as share of income by Total, Metropolitan Area and Rest

Road Length Contiguity				
Equivalent Variation	Mean	Median	90% HPD Interval	
			Lower	Upper
M. Area	0.126%	0.141%	0.006%	0.209%
Rest	0.940%	0.655%	0.257%	2.006%
Total	0.874%	0.630%	0.005%	1.913%

Source: Author's calculations

Main Results

Table 6: Equivalent Variation as share of income for representative municipalities

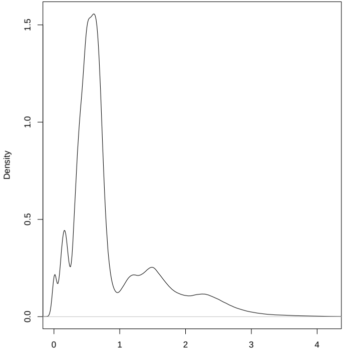
Road Length Contiguity				
Municipality	Mean	Median	90% HPD Interval	
			Lower	Upper
Medellín	0.137%	0.139%	0.081%	0.194%
Barbosa	0.008%	0.008%	0.006%	0.009%
San José de la Montaña	0.313%	0.313%	0.307%	0.319%
Santa Fe de Antioquia	2.297%	2.297%	1.956%	2.619%

Source: Author's calculations

Low income households expend on average 1.13%, 1.74% and 4.65% of their income in pension, health care and education.

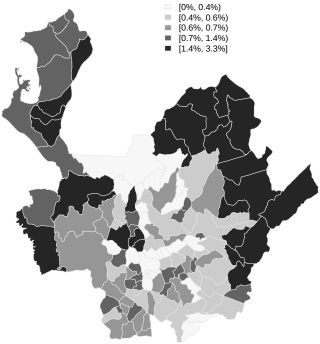
Main Results

Figure 4: Equivalent Variation Posterior Distribution



Main Results

Figure 5: Equivalent Variation Posterior Distribution



Outline

Conclusions

- Introduced spatial random effects into an endogenous Bayesian framework with simultaneous equations and deduced the complete conditional posterior distributions.
- Controlled for spatial dependence, endogenous regressors, weak instruments and unobservable heterogeneity simultaneously.
- Estimated a system of equations for electricity and find the average price, income, substitute and urbanization rate demand elasticities to be -0.88, 0.30, 0.12 and 0.57, respectively.

Conclusions

- Estimated the Equivalent Variation for the total of the province to be 0.87% in average and 0.63% in median.
- The welfare gains of the Metropolitan Area amount only to 0.13%.
- Municipalities that are not part of the Metropolitan Area gained in average 0.94%
- The less urban and poorest municipalities, increased their welfare well above 2% of the initial income.
- The complete analysis would be extremely difficult to handle, were it not for the Bayesian approach that allowed us great flexibility and structure.

References I

Acton, J. P. and Mitchell, B. M. (1983). *Welfare analysis of electricity rate changes*. RAND Corporation.

Anselin, L. (1988). *Spatial econometrics: methods and models*, volume 4. Springer.

Anselin, L. (2003). Spatial Externalities, Spatial Multipliers and Spatial Econometrics. *International Regional Science Review*, 26(2):153–166.

Banerjee, S., Carlin, B. P., and Gelfand, A. E. (2004). *Hierarchical modeling and analysis for spatial data*. Chapman & Hall / CRC Press.

Berger, J. O. (1985). *Statistical decision theory and Bayesian analysis*. Springer.

Besag, J., York, J., and Mollié, A. (1991). Bayesian image restoration, with two applications in spatial statistics. *Annals of the Institute of Statistical Mathematics*, 43(1):1–20.

References II

Chakraborty, A., Beamontec, M. A., Gelfand, A. E., Alonso, M. P., Gargallo, P., and Salvador, M. (2013). Spatial interaction models with individual-level data for explaining labor flows and developing local labor markets. *Computational Statistics and Data Analysis*, 58(1):292–307.

Cressie, N. (1993). *Statistics for spatial data*. Jhon Willey & Sons, INC, United States, revised edition.

Crukker, D. M., Egger, P., and Prucha, I. R. (2013). On two-step estimation of a spatial autoregressive model with autoregressive disturbances and endogenous regressors. *Econometric Reviews*, 32(5-6):686–733. Forthcoming.

Darmofal, D. (2009). Bayesian spatial survival models for political event processes. *American Journal of Political Science*, 53(1):241–257.

Dodonov, B., Opitz, P., and Pfaffenberger, W. (2004). How much do electricity tariff increases in Ukraine hurt the poor? *Energy Policy*, 32(7):855–863.

References III

Fingleton, B. and Gallo, J. L. (2008). Estimating spatial models with endogenous variables a spatial lag and spatially dependent disturbances: finite sample properties. *Papers in Regional Science*, 87(3):319–340.

Gomez-Lobo, A. (1996). The welfare consequences of tariff rebalancing in the domestic gas market. *Fiscal Studies*, 17(4):49–65.

Greenberg, E. (2008). *Introduction to Bayesian Econometrics*. Cambridge University Press.

Hausman, J. A. (1981). Exact consumer's surplus and deadweight loss. *American Economic Review*, 71(4):662–676.

Kelejian, H. H. and Prucha, I. R. (2004). Estimation of simultaneous systems of spatially interrelated cross sectional equations. *Journal of Econometrics*, 118:27–50.

References IV

Kelsall, J. E. and Wakefield, J. C. (1999). Discussion of 'Bayesian models for spatially correlated disease and exposure data' by best et al. In Bernardo, J. M., Berger, J. O., Dawid, A. P., and Smith, A. F. M., editors, *Bayesian Statistics 6: Proceedings of the Sixth Valencia International Meeting*, page 151. Oxford University Press.

LeSage, J. P. (1997). Bayesian estimation of spatial autoregressive models. *International Regional Science Review*, 20:113–119.

LeSage, J. P. (2000). Bayesian estimation of limited dependent variable spatial autoregressive models. *Geographical Analysis*, 32(1):19–35.

LeSage, J. P., Fischer, M. F., and Scherngell, T. (2007). Knowledge spillovers across Europe: Evidence from a Poisson spatial interation model with spatial effects. *Papers in Regional Science*, 86(3):393–422.

References V

LeSage, J. P. and Llano, C. (2013). A spatial interaction model with spatially structured origin and destination effects. *Journal of Geographical Systems*, 15:265–289.

LeSage, J. P. and Pace, R. K. (2009). *Introduction to Spatial Econometrics*. Taylor & Francis, Boca Raton.

Liu, X. and Lee, F.-L. (2013). Two stage least squares estimation of spatial autoregressive models with endogenous regressors and many instruments. *Econometric Reviews*. Forthcoming.

Lundgren, J. (2009). Consumer welfare in the deregulated Swedish electricity market. *Frontiers in Finance and Economics*, 6(2):101–119.

Parent, O. and LeSage, J. P. (2008). Using the variance structure of the conditional autoregressive spatial specification to model knowledge spillovers. *Journal of Applied Econometrics*, 23:235–256.

References VI

Rey, S. J. and Boarnet, M. G. (2004). A taxonomy of spatial econometric models for simultaneous equation systems. In Anselin, L., Florax, R. J., and Rey, S. J., editors, *Advances in Spatial Econometrics: Methodology, Tools and Applications*, pages 99–119. Springer.

Ripley, B. (2004). *Spatial Statistics*. Jhon Wiley & Sons, United States, second edition.

Ruijs, A. (2009). Welfare and distribution effects of water pricing policies. *Environmental and Resource Economics*, 43(2):161–182.

Seya, H., Tsutsumi, M., and Yamagata, Y. (2012). Income convergence in Japan: A Bayesian spatial Durbin model approach. *Economic Modelling*, 29:60–71.

References VII

Smith, T. E. and LeSage, J. P. (2004). A bayesian Probit model with spatial dependences. In LeSage, J. P. and Pace, R. K., editors, *Advances in Econometrics: Volume 18: Spatial and Spatiotemporal Econometrics*, pages 127–160. Elsevier Ltd.

Thomas, A., Best, N., Lunn, D., Arnold, R., and Spiegelhalter, D. (2007). *GeoBUGS User Manual version 1.3*.

Varian, H. R. (1992). *Microeconomic Analysis*. W.W. Norton & Company, New York, Third edition.

Wall, M. (2004). A close look at the spatial structure implied by the CAR and SAR models. *Journal of Statistical Planning and Inference*, 121:311–324.

You, J. S. and Lim, S. (2013). Welfare effects of nonlinear electricity pricing. *Submission for the 37th International Association for Energy Economics International Conference*.

Zellner, A. (1996). *An Introduction to Bayesian Inference in Econometrics*. Wiley, New York, Second edition.